Comparison between Zubieta Model of Supercapacitors and their Real Behavior

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Abstract: The supercapacitor is a relatively young electronic component which bases its operation on the principle of the Helmholtz double layer of charge formed on the surface of an activated carbon (with an extremely high equivalent surface) and an electrolyte which allows both a short distance between the electric layers as well as a very large useful area.

For optimal use of an EDLC (Electrochemical Double Layer Capacitor), it is necessary to know the time behavior, specific electrical characteristics (discharge current, charge current, voltage variation at the terminals, equivalent series resistance, and especially leakage current (self-discharge current), as well as behavior over a long time – days or even months - under electric charge at different voltages, etc.) and the influence of environmental operating conditions (temperature, humidity, vibration, etc.). For modeling in more detail the behavior of an EDLC in time and voltage a comparison between the Zubieta model and actual behavior was performed. The research is focused on the accuracy of the Zubieta model related to the real behavior of supercapacitors.

Keywords: Supercapacitor, EDLC, Zubieta, comparison

I. INTRODUCTION

The approximation of the operation of a supercapacitor proposed by Zubieta (see Fig. 1) does not fully highlight its experimental behavior. As shown in Fig. 1 in the first branch, named immediate branch, the capacitor \( C_{im} \) highlights the voltage dependence of the double-layer. Moreover, the leakage current is modeled using the \( R_p \) resistor.

For calculating the parameters of the immediate branch, the supercapacitor is charged with a constant current \( I_{ch} \).

At \( t_1 \) (a few seconds after charging starts) we measure voltage \( V_1 \). Knowing the voltage value, and the current value respectively the resistance value for the immediate branch can be determined:

\[
R_{im} = \frac{V_1}{I_{ch}}
\]  

(1)

After \( t_2 \), the voltage that is measured at the terminals of the supercapacitor will increase by \( \Delta V \), and the capacity \( C_{im} \) will be calculated according to the formula below:

\[
C_{im} = \frac{I_{ch} \cdot t_2 - t_1}{\Delta V}
\]  

(1)

Observation: \( t_2 \) was chosen so that \( \Delta V = 50 \text{ mV} \).

The time moment \( t_3 \) is when the voltage reaches the maximum value specified by the manufacturer in the supercapacitor’s data sheet, and the power source is turned off. After this stage the EDLC begins the discharge process and that moment represents the time moment \( t_4 \) and the
measured voltage will be $V_4$. Having all this information the value of $C_{i1}$ can be computed:

$$C_{i1} = \frac{2}{V_4} \left( \frac{\Delta V}{I_{ch}(t_4 - t_1)} - C_{i0} \right)$$  \hspace{1cm} (2)

With this information the total load of the supercapacitor can be calculated:

$$Q_{tot} = I_{ch}(t_4 - t_1)$$  \hspace{1cm} (3)

Having calculated all the parameters of the immediate branch, the parameters of the delayed branch can now be determined.

We will wait for a longer time $t_5$ until the voltage drops with $\Delta V_2$ against to the value of voltage $V_4$. $t_5$ represents the time necessary to transfer the electrical current from the immediate branch to the timed branch. The resistance value $R_d$ is calculated with:

$$R_d = \frac{(V_4 - \frac{\Delta V_2}{2})(t_5 - t_6)}{(C_{i0} + C_{i1}(V_4 - \frac{\Delta V_2}{2}))\Delta V_2}$$  \hspace{1cm} (4)

For the calculation of the timed branch’s capacity, voltage $V_5$ is measured at $t_6$, where $t_6 = 3t_5$:

$$C_d = \frac{Q_{tot}}{V_5} - \left( C_{i0} + C_{i1} \frac{V_5}{2} \right)$$  \hspace{1cm} (5)

To calculate the parameters of the long-term branch the following procedure should be followed: to determine resistance $R_L$ we will wait for a longer time period noted with $t_7$ until the voltage drops with $\Delta V_3$ against the value of voltage $V_5$:

$$R_L = \frac{(V_5 - \frac{\Delta V_3}{2})(t_7 - t_6)}{(C_{i0} + C_{i1}(V_5 - \frac{\Delta V_3}{2}))\Delta V_3}$$  \hspace{1cm} (6)

Long-term capacity is calculated after 30 minutes, note $t_8$ since this is considered a long enough time for the charge to be fully distributed the long-term branch. This is when the voltage on each branch, will be balanced and equal to $V_6$.

The capacity’s value $C_L$ will be determined according to the formula:

$$C_L = \frac{Q_{tot}}{V_6} - \left( C_{i0} + C_{i1} \frac{V_6}{2} - C_d \right)$$  \hspace{1cm} (7)

To perform simulations a circuit was developed allowing the charging and discharging of supercapacitors. Each supercapacitor must be charged up to maximum voltage without exceeding it. The electronic schematic from Fig. 2 realizes the voltage protection of the supercapacitors used. When the voltage reaches the maximum value specific to each supercapacitor (2.5 V, 5.4 V respectively), the transistors open and thus fulfill their role. The LED acts as a “witness” and specifies that the transistors were opened when the supercapacitores peaked at the maximum voltage value.

![Fig. 2. The electronic scheme used for simulation at 2.5 V.](image)

### III. Measurements And Results

In this work measurements were conducted for two type of supercapacitors with capacitances of 22F and 5F. The rated voltages for these supercapacitors are 2.5 V, respectively 5.4 V. Initially the supercapacitors were charged with a current of 100 mA and the rules of the Zubita model were applied. Subsequently, the supercapacitors were discharged with the same current using an active sink. All the data was saved on a PC using specialized software and with
this data all the equivalent parameters of the Zubieta model for each branch were calculated. A model based on the obtained values was implemented and simulations conducted.

In Fig. 3 the measurements setup is presented and on the PC we can see the working software.

The values obtained for each branch calculated with the formulas of Zubieta model for the used supercapacitors, are show in the table 1.

The Zubieta model was implemented on the electronic scheme (see Fig. 2) with the values of equivalent parameters that were calculated and are presented in table1 and table 2. After this the simulation of electronic scheme for each supercapacitor was realized.

The comparison between the Zubieta model of supercapacitors (22 F, respectively 5 F) and their real behavior is presented in the figures below:

<table>
<thead>
<tr>
<th>TABLE 1 THE VALUES OF THE PARAMETERS FOR 22F AND 5V</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUPERCAPACITORS</td>
</tr>
<tr>
<td>Immediate branch</td>
</tr>
<tr>
<td>R_i</td>
</tr>
<tr>
<td>C_i0</td>
</tr>
<tr>
<td>C_i1</td>
</tr>
<tr>
<td>Q_tot</td>
</tr>
<tr>
<td>Delayed branch</td>
</tr>
<tr>
<td>R_d</td>
</tr>
<tr>
<td>C_d</td>
</tr>
<tr>
<td>Long term branch</td>
</tr>
<tr>
<td>R_L</td>
</tr>
<tr>
<td>C_L</td>
</tr>
</tbody>
</table>

Fig. 4. The simulation (green) and the measurement (blue) for the 22 F/2.5 V supercapacitor.
We can see in the Fig. 4 and in Fig. 5 that the approximations of the operation of the monitored supercapacitors proposed by Zubieta does not fully highlight its experimental behavior.

IV. CONCLUSIONS

Zubieta assumes that in the model with three branches only the capacity of immediate branch $C_{i0}$ is dependent on the voltage which can lead to errors for low voltages.

Because for the long-term branch is needed only 30 minutes that matter, this model can’t be applied to the supercapacitors with higher capacities, tens or hundreds Farads. A solution to this problem would be extending the Zubieta model on multiple branches to monitor the EDLC’s operation for a long time, tens of hours or even days.

For the simulation of each supercapacitor different voltage values were needed and for this we can use different electronic schemes.

After the comparison between the Zubieta model of supercapacitors and their real behavior we can see that differences arise because it does not monitors the phenomena of supercapacitors self-discharged.

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